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60

ON PSEUDOSUPERSYMMETRIC OSCILLATORS AND REALITY OF RELATIVISTIC ENERGIES FOR VECTOR MESONS

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Abstract

Specific oscillators - hereafter called pseudosupersymmetric oscillators - appear as interesting nonrelativistic concepts in connection with the study of relativistic vector mesons interacting with an external constant magnetic field when the real character of the energy eigenvalues is required as expected. A new pseudosupersymmetric quantum mechanics can then be developed and the corresponding pseudosupersymmetries can be pointed out.

1. Introduction

There are two old problems which appear when we study the interaction of relativistic vector mesons (spin one particles with nonzero rest mass) with external constant magnetic fields chosen, in particular, as directed along the z-axis, i.e. $\vec{B} = (0,0,B)$. The first one which will be of special interest here is mainly connected to the energy eigenvalue problem subtended by the inclusion of an anomalous moment coupling inside the relativistic equation of motion ensuring that the spin 1-boson has a gyromagnetic ratio value of $g = 2$. A survey of such a question has recently been presented in the Daicic-Frankel study^[1] where further references can also be found. We will refer to that paper in order to save place here. The second problem is concerned with the fulfilment of the causality principle through the corresponding wave equation

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describing such an interaction. Its discussion can be referred to another contribution^[2] where the so-called method of characteristics^[3] plays the prominent role. Let us only comment on the fact that our final equation will satisfy all the requirements needed by the causality principle but it will not be discussed here, so that we can polarize our attention mainly on the first problem.

After a short remark pointing out the first difficulty (§2), we want to insist on our understanding of the so-called parasupersymmetric quantum mechanics (PSSQM)^[4,5] for motivating our way of eliminating the above problem (§3) and for studying the new (nonrelativistic) formulation and Hamiltonian that we get in that way (§4). Besides the usual bosons, we are led to the introduction of a new kind of fermions that we call "pseudofermions" (§5) and to new symmetries that we call "pseudosupersymmetries" as it will be clear in the following by comparison with well known supersymmetries^[6] and parasupersymmetries^[4,5].

2. On the reality of (relativistic) energy eigenvalues

By exploiting the remarkable Johnson-Lippmann contribution^[7] developed for spin $\frac{1}{2}$ -particles, it is easy^[1] to decompose the spin 1-formalism in a z-part associated with the so-called $H_{//}$ and, in a (x,y)-part, associated with the so-called H_{\perp} , the latter being readily studied through 1-dimensional harmonic oscillator characteristics. Then, the energy eigenvalue problem for vector mesons leads to information such that

$$E^2 = 1 + e B \left(n + \frac{1}{2} \right) - 2 e B s \quad (2.1)$$

where e is the charge of the vector meson, n the Landau-level quantum number ($n = 0, 1, 2, \dots$) and s the eigenvalues of the third component of the spin operator ($s = 0, \pm 1$), when we have chosen $\hbar = 1$, $m = 1$, $c = 1$ and defined the angular frequency $\omega = e B$ of the resulting harmonic oscillator. In eq.(2.1), the first term in the righthand side corresponds to the relativistic rest mass term, the second one to the original discussion of H_{\perp} and the third one to the presence of an anomalous magnetic moment coupling^[1]. Such an equation evidently permits

$$E^2 < 0 \quad (2.2)$$

when $eB > 1$, $n = 0$, $s = 1$, so that we are dealing with the problem of possible complex energy eigenvalues for intense magnetic fields of critical magnitudes.

3. Some observations from PSSQM

PSSQM has been proposed by Rubakov and Spiridonov^[4] and slightly modified by us^[5]. Both approaches consider the superposition of bosons and parafermions^[8] of order 2 and lead to 3-fold degeneracies in the energy spectrum. But, if, in the Rubakov-Spiridonov context^[4], the groundstate is characterized by a negative energy eigenvalue, our groundstate has a null energy eigenvalue^[9]. We have thus pointed out that the relativistic result (2.2) could have a direct connection with the nonrelativistic Rubakov-Spiridonov approach and that, consequently, we could handle the problem by exploiting our approach and its relativistic counterpart excluding results such as those given by eqs.(2.1) and (2.2).

In fact, such a method has recently been developed by one of us^[10] by following strictly our PSSQM-context^[5]. Here, we want, in a certain complementary way, to center our attention on new harmonic oscillatorlike characteristics ensuring the reality of the energy eigenvalues.

4. To a new nonrelativistic Hamiltonian

We have modified the relativistic formulation of vector mesons interacting with our \vec{B} -magnetic field in such a way that we get a six-component Klein-Gordon type equation of the form

$$p_0^2 \chi(x) = (1 + \pi_1^2 + \pi_2^2 + p_3^2 + eB - 2eB \Sigma_3) \chi(x) \quad (4.1)$$

where

$$\pi_i = p_i - eA_i, \quad i = 1, 2, \quad (4.2)$$

$$A_1 = -\frac{1}{2}By, \quad A_2 = \frac{1}{2}Bx, \quad (4.3)$$

$$\Sigma_3 = \begin{pmatrix} s_3 & 0 \\ 0 & s_3 \end{pmatrix}, \quad s_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (4.4)$$

This 6 by 6 formulation leads to a bounded energy spectrum ($E_0 \geq 0$) with 3-fold degeneracies and admits a nonrelativistic (NR) limit characterized by the Hamiltonian

$$H_{NR} = \frac{1}{2} (\pi_1^2 + \pi_2^2) + \frac{\theta B}{2} (1 - 2 s_3). \quad (4.5)$$

Such developments solve our (relativistic) problem connected with eqs.(2.1) and (2.2). They also point out a new Hamiltonien (4.5) which, now, has to be visited.

5. From this new relativistic Hamiltonian : the appearance of "pseudofermions"

From supersymmetric^[6] as well as parasupersymmetric^[4,5] considerations, we immediately observe that the Hamiltonian (4.5) is made of two contributions : the first term is a purely bosonic part constructed in terms of even bosonic operators π_1 and π_2 while the second term looks like a purely "fermionic" part constructed in terms of odd "fermionic" operators called hereafter A and B. In fact, we propose to construct new charges

$$Q_1 = A \pi_1 + B \pi_2, \quad Q_2 = -B \pi_1 + A \pi_2, \quad (5.1)$$

where

$$A = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & 0 & 1+i \\ 0 & 0 & -1+i \\ 1-i & -1-i & 0 \end{pmatrix}, \quad B = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & 0 & 1-i \\ 0 & 0 & 1+i \\ 1+i & 1-i & 0 \end{pmatrix}. \quad (5.2)$$

The matrices A and B are Hermitean (so that the charges also are) and have a manifestly odd character. It is straightforward to show that, with $i, j = 1, 2$, we have

$$Q_i^3 = Q_i H_{NR}, \quad [Q_i, H_{NR}] = 0, \quad (5.3a)$$

$$Q_i^2 Q_j = Q_j Q_i^2 = -Q_i Q_j Q_i = Q_j H_{NR}, \quad i \neq j. \quad (5.3b)$$

Such a structure is neither a Lie superalgebra^[11], nor a Lie parasuperalgebra^[12]. It is more clearly characterized when we refer to the two charges

$$Q = c(Q_1 - iQ_2), \quad Q^\dagger = c(Q_1 + iQ_2), \quad c \in \mathbb{R}. \quad (5.4)$$

Indeed they lead to the structure relations

$$Q^2 = 0, \quad Q^{\dagger 2} = 0, \quad [H_{NR}, Q] = [H_{NR}, Q^\dagger] = 0, \quad (5.5)$$

$$Q Q^\dagger Q = 4c^2 Q H_{NR}, \quad Q^\dagger Q Q^\dagger = 4c^2 Q^\dagger H_{NR},$$

already obtained by Semenov and Chumakov^[13] as possible ones associated with the discussion of 3-level systems. By searching for the charges (5.4) in terms of annihilation and creation (oscillatorlike) operators, we define

$$Q = \frac{1}{2} (A + iB)(\pi_1 - i\pi_2) = \sqrt{\omega} b a^\dagger, \quad Q^\dagger = \sqrt{\omega} b^\dagger a, \quad (5.6)$$

with
$$a = \frac{1}{\sqrt{2}\omega} (\pi_1 + i\pi_2) \quad (5.7)$$

as usual. We then get information on our "fermionic" operators b and b^\dagger such that

$$b^2 = b^{\dagger 2} = 0, \quad b b^\dagger b = b, \quad b^\dagger b b^\dagger = b^\dagger. \quad (5.8)$$

These relations mean that the corresponding particles are bosons (see eq.(5.7)) and "a new kind of fermions" (see eq.(5.8)) that we call "pseudofermions". The first equalities in eqs.(5.8) corresponding to nilpotencies show that they are not far to satisfy the Pauli principle but the last equalities say that they are not at all usual fermions. Moreover, we can prove that they are neither parafermions^[8], nor quons^[14], nor orthofermions^[15].

For $c = 1$ or $\frac{1}{2}$, eqs.(5.4) and (5.5) appear to be compatible with those of PSSQM developed by Rubakov-Spiridonov^[4] or by us^[5], respectively. Moreover, the corresponding structure relations of supersymmetric developments^[6] imply the eqs.(5.5).

These properties suggest that our symmetries (evidently called "pseudosupersymmetries") are, in a certain sense, "between" super- and parasupersymmetries. We have thus the basic ingredients of a new tool that we call "pseudosupersymmetric quantum mechanics" which could be developed in terms of two "pseudopotentials" W_1 and W_2 .

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